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Reduction of thermal stresses by developing two-dimensional functionally graded materials

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Abstract

Modern aerospace shuttles and craft are subjected to super high temperatures, that have variation in two or three directions, which need to introduce new materials that can stand with such applications. Therefore, in the present work a two-dimensional functionally graded materials, 2D-FGM, are introduced to withstand super high temperatures and to give more reduction in thermal stresses. The suitable functions that can represent volume fractions of the introduced 2D-FGM are proposed. Then the rules of mixture of the 2D-FGM are derived based on the volume fractions of the 2D-FGM and the rules of mixture of the conventional FGM. The introduced volume fractions and rules of mixture for 2D-FGM were used to calculate the thermal stresses in 2D-FGM plate. Comparison between 2D-FGM and conventional FGM was carried out and showed that 2D-FGM has high capability to reduce thermal stresses than the conventional FGM.

Keywords: FGM; Rules of mixture; Volume fractions; 2D-FGM; Thermal stresses

1. Introduction

The high level of operating temperature involved in many industrial machine elements, such as space shuttles, combustion chambers, nuclear plants and ovens, requires effective high temperature resistant materials to improve the strength of such machine elements. In recent years, the concept of *functionally graded material* (FGM) has been introduced as a thermal barrier material. FGMs were adopted for the development of structural components subjected to severe thermal loading. Day by day FGMs prove their high capability as high temperature resistant material and quietly gain this position. Many authors, such as; Noda and Tsuji (1991), Arai et al. (1991), Tang et al. (1993), Erdogan and Wu (1996) and Jin and Batra (1996), analyzed FGM problems in different cases with and without crack under thermal and mechanical loads. Their results were very exciting and enhanced their aims in the development of new temperature resisting materials.

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During the investigation of the FGM problems different functions for the continuous gradation of the material properties were considered. In analytical solution of FGM problems, under thermal and mechanical loads, exponential functions for continuous gradation of the material properties were considered, see for example; Erdogan et al. (1991), Erdogan and Wu (1997), Choi (1996), Jin and Noda (1994), Noda and Jin (1993, 1995) and Wang et al. (2000), as: $E = E_0 e^{\beta x}$, $v = v_0 e^{\beta x}$, $k = k_0 e^{\delta x}$ and $\alpha = \alpha_0 e^{\gamma x}$, where E: Young's modulus, v: Poisson's ratio, α: coefficient of linear thermal expansion, k: heat conductivity and E_0, v_0, α_0, k_0, β, γ and δ are material constants. Exponential functions for representation of the material properties usually facilitate the analytical solution but don't give real representation for material properties, expect for the upper and lower surfaces of FGM. The most realistic way for representation of the continuous gradation of the material properties of FGM is the volume fractions and rules of mixture. The use of the volume fractions and rules of mixture complicate the analytical solution of the FGM problems and may make it impossible. The use of finite element method in such problems is the most effective tools to overcome such difficulties. Many authors, such as Fuchiyama et al. (1993), Noda (1997), Sumi and Sugano (1997), Hassab-Allah and Nemat-Alla (2002) and Fujimoto and Noda (2001), analyzed FGM problems, in different cases with and without crack, using the volume fractions and rules of mixture.

Steinberg (1986) showed the variations of the temperature at various places on the outer surface, of the new aerospace craft when the plane is in sustained flight at a speed of Mach 8 and altitude of 29 km. The temperature on the outer surface of such a plane ranges from 1033 K along the top of the fuselage to 2066 K at the nose and from outer surface temperature to room temperature inside the plane. Such kind of aerospace craft added a new challenge to introduce and develop more high temperature resistant materials that can stand with high external temperatures that have variation in two or three directions. In the design requirement of such problem Callister (2001) suggested to design several different thermal protection materials that satisfy the required criteria for a specific region of the spacecraft surface. Callister design has the same drawbacks of the composite layers, cracking, separation through layers interface and low mechanical strength, that were overcome by functionally graded materials. Recently, Colombia space shuttle was loosed in a catastrophic break up. According to International Herald Tribune (2003) the speculation of such failure is that some kind of structural damage took place—perhaps caused by outer surface insulation that fell loose when the Columbia lifted off. Also, Hindustan Times (2003) reported that, damage to the space shuttle's protective thermal tiles is emerging as a key focus of the probe into the Colombia tragedy.

Conventional functionally graded material may also not be so effective in such design problems since all outer surface of the body will have the same composition distribution. Since, temperature distribution in such advanced machine element changes in two or three directions. Therefore, if the FGM has two-dimensional dependent material properties, more effective high-temperature resistant material can be obtained. In other words it is necessary to add a material that has more strength to the thermal stresses in the places that have maximum values of the thermal stresses or the places where yielding may occur. Based on such fact a *two-dimensional*, 2D, FGM whose material properties are two-directional dependent is introduced. 2D-FGM takes a great interest as higher order theory for the thermo-elastic response of composite materials in order to achieve more relaxation of the thermal stresses and their intensity factors. The two-dimensional models of the material properties of FGM increase the number of thermal/mechanical non-homogeneous parameters. The wide variety of thermal/mechanical non-homogeneous parameters will increase the ability of reducing thermal stresses and their concentration factors.

Recently, few authors investigated 2D-FGM. Dhaliwal and Singh (1978) solved the equation of equilibrium for non-homogeneous isotropic elastic solid under shearing force in rectangular Cartesian coordinates as well as in cylindrical polar coordinates. The modulus of rigidity of their considered material was varied exponentially in lateral and vertical directions. They considered the problem of determining the state of stresses in an infinite non-homogenous elastic medium containing a Griffith crack under shearing force. Clements et al. (1997) introduced the solution of the equations of antiplane inhomogeneous elasticity when the shear modulus varied continuously with two Cartesian coordinates. A particular crack problem was considered for a certain multi-parameter form for the shear modulus. Aboudi et al. (1996a,b), studied thermo-elastic/plastic theory for the response of materials functionally graded in two directions. Their studies circumvented the problematic use of the standard micro-mechanical approach, based on the concept of a representative volume element, commonly employed in the analysis of functionally graded composites by explicitly couple the local (micro-structural) and global (macro-structural) responses. The response of symmetrically laminated plates subjected to temperature change in one-dimension was investigated. It was possible to reduce the magnitude of thermal stress concentrations by a proper management of the microstructure of the composite. Nemat-Alla and Noda (1996a,b, 2000) and Nemat-Alla et al. (2001) analyzed the crack problem in semi-infinite and finite FGM plate with bi-directional coefficient of thermal expansion under one and two-dimensional steady thermal loads. They showed that the thermal stresses and the thermal stress intensity factors could be decreased by the proper selection of the mechanical and thermal non-homogeneous parameters. Cho and Ha (2002) optimize the volume fractions distributions of FGM for relaxing the effective thermal stresses. They obtained the optimal volume fractions distribution in two directions for the FGM. The obtained optimum volume fractions have not continues, direct or function, representation as conventional FGM. It looks like a random distribution, which is very difficult to represent or simulate FGM that have continues variations of the composition. Also, their obtained optimum volume fractions was investigated under steady thermal loading, cooling to 300 K uniform temperature from uniform initial temperature of 1000 K. Such kind of thermal loading can be resisted by conventional FGM.

From the forgoing review of literature, one can see that several works have been carried out to investigate the thermal stresses and the thermal stress intensity factors in 2D-FGM under thermal and mechanical loading. The material properties were considered to be exponential functions in two-directions. Unfortunately, the rules of mixture and the volume fractions relations that can represent the 2D-FGM are not available. The main objective of the current investigation is to introduce the rules of mixture and the volume fractions relations that can represent the 2D-FGM. This is very important in order to characterize the behavior of the 2D-FGM under 2D and 3D-thermal loading. The developed volume fractions and rules of mixture relations of the 2D-FGM will be used to calculate the thermal stresses in a 2D-FGM plate. Then a comparison of the thermal stresses in 2D-FGM plate with the conventional FGM plate will be carried out.

2. Volume fractions and rules of mixture of FGM

Consider a plate of FGM with porosity that functionally graded from ceramic and metal. The volume fractions of FGM with porosity can be represented as:

$$V_{\mathrm{m}} = (x/t)^{m} \tag{1}$$

$$V_{\mathrm{c}} = (1 - V_{\mathrm{m}}) \tag{2}$$

where V_{m}, V_{c}, x and t are volume fraction of the metal, volume fraction of ceramic, Cartesian coordinate x and plate thickens respectively.

Also, m is a non-homogenous parameter that control the composition variations through the thickens. If $m = 1$ the variations of the composition of ceramics and metal are linear. The composition is metal-rich when $m < 1$ and metal poor when $m > 1$.

Where the porosity p of the FGM is represented as:

$$p = A\left(\frac{x}{t}\right)^{n}\left[1 - \left(\frac{x}{t}\right)^{z}\right] \tag{3}$$

where

$$\frac{\left(\frac{n+z}{n}\right)^n}{1-\left(\frac{n}{n+z}\right)^z} \geqslant A \geqslant 0 \tag{4}$$

n and z are arbitrary constants.

The effective values of the material properties for FGM, with porosity and continuously graded profile, are determined by employing the suspended spherical grain model (see Kerner, 1956; Kingery et al., 1976). It was derived based on the assumption that the granular phase is in a matrix phase. The following relations give the rules of mixture for the different thermal and mechanical properties.

$$\lambda = \frac{\lambda_0\{(1-p^{2/3})\lambda_0 + p^{2/3}\lambda_a\}}{p^{1/3}\lambda_a + (1-p^{1/3})\{(1-p^{2/3})\lambda_0 + p^{2/3}\lambda_a\}} \tag{5}$$

$$E = \frac{E_0(1-p)}{1+p(5+8v)(37-8v)/\{8(1+v)(23+8v)\}} \tag{6}$$

$$\alpha = \alpha_0 \tag{7}$$

$$v = v_0 \tag{8}$$

$$\rho = \rho_0(1-p) + \rho_a p \tag{9}$$

$$C = C_0(1-p) + C_a p \tag{10}$$

$$\sigma_Y = \sigma_{Y1}V_1 + \sigma_{Y2}V_2 \tag{11}$$

where λ, E, α, v, ρ, C and σ_Y are thermal conductivity, modulus of elasticity, coefficient of thermal expansion, Poisson's ratio, density, specific heat and yield stress respectively. Also,

$$\lambda_0 = \lambda_c\left[1 + \frac{3(\lambda_m - \lambda_c)V_m}{3\lambda_c V_m + (\lambda_m + 2\lambda_c)(1-V_m)}\right]$$

$$E_0 = E_c\left[\frac{E_c + (E_m - E_c)V_m^{2/3}}{E_c + (E_m - E_c)(V_m^{2/3} - V_m)}\right]$$

$$\alpha_0 = \alpha_c V_c + \alpha_m V_m + \frac{V_m V_c(\alpha_m - \alpha_c)(K_m - K_c)}{K_c V_c + K_m V_m + (3K_m K_c/4G_c)}$$

$$K_c = \frac{E_c}{3(1-2v_c)}$$

$$K_m = \frac{E_m}{3(1-2v_m)}$$

$$G_c = \frac{E_c}{2(1+v_c)}$$

$$G_{\mathrm{m}} = \frac{E_{\mathrm{m}}}{2(1+v_{\mathrm{m}})}$$

$$C_0 = \frac{C_{\mathrm{m}} V_{\mathrm{m}} \rho_{\mathrm{m}} + C_{\mathrm{c}} V_{\mathrm{c}} \rho_{\mathrm{c}}}{V_{\mathrm{m}} \rho_{\mathrm{m}} + V_{\mathrm{c}} \rho_{\mathrm{c}}}$$

$$v_0 = v_{\mathrm{m}} V_{\mathrm{m}} + v_{\mathrm{c}} V_{\mathrm{c}}$$

$$\rho_0 = \rho_{\mathrm{m}} V_{\mathrm{m}} + \rho_{\mathrm{c}} V_{\mathrm{c}}$$

where K and G are the bulk's modulus and modulus of rigidity respectively. Subscripts m, c and a indicate metal, ceramic and air respectively.

3. Volume fractions of 2D-FGM

2D-FGM is made of continuous gradation of three distinct material phases at least one of them is ceramics and the others are metal alloy phases. 2D-FGM is fabricated in such a way that the volume fractions of the constituents are varied continuously in a predetermined composition profile.

Now let us discuss the volume fractions and porosity of the 2D-FGM at any arbitrary point A on the 2D-FGM plate shown in Fig. 2. Firstly, the volume fractions of point A may be treated as one-dimensional FGM that consists of two volume fractions V_3 and V_s, where V_s is a mixture of V_1 and V_2. Therefore V_3 and V_s can be proposed as:

$$V_3 = \left(\frac{y}{t}\right)^{m_y} \tag{12}$$

$$V_s = 1 - \left(\frac{y}{t}\right)^{m_y} \tag{13}$$

$$p_y = A_y \left(\frac{y}{t}\right)^{n_y} \left[1 - \left(\frac{y}{t}\right)^{z_y}\right] \tag{14}$$

where

$$\frac{\left(\dfrac{n_y + z_y}{n_y}\right)^{n_y}}{1 - \left(\dfrac{n_y}{n_y + z_y}\right)^{z_y}} \geqslant A_y \geqslant 0 \tag{15}$$

and p_y is the porosity between V_3 and V_s.

Also, the volume fractions V_1 and V_2 can be obtained by considering the lower surface of the 2D-FGM plate as one-dimensional FGM then V_1, V_2 and porosity p_x can be expressed as:

$$V_2 = \left[1 - \left(\frac{y}{t}\right)^{m_y}\right] \left(\frac{x}{l}\right)^{m_x} \tag{16}$$

$$V_1 = \left[1 - \left(\frac{y}{t}\right)^{m_y}\right] \left[1 - \left(\frac{x}{l}\right)^{m_x}\right] \tag{17}$$

$$p_x = A_x \left(\frac{x}{l}\right)^{n_x} \left[1 - 2\left(\frac{y}{t}\right)^{z_y} + \left(\frac{y}{t}\right)^{2z_y}\right] \left[1 - \left(\frac{x}{l}\right)^{n_x}\right] \tag{18}$$

where

$$\frac{\left(\frac{n_x+z_x}{n_x}\right)^{n_x}}{V_2+V_1-\left(\frac{n_x}{n_x+z_x}\right)^{z_x}} \geqslant A_x \geqslant 0 \tag{19}$$

and subscripts 1, 2 and 3 denote material 1, material 2 and material 3 of the basic constituents. Also, m_y and m_x are non-homogenous parameters that represent the composition distributions of the basic constituents materials in y- and x-directions and A_y, A_x, n_x, z_x n_y, and z_y are arbitrary parameters that control the porosity.

From proposed volume fractions at any point, the volume fractions of the three basic constituents materials on each boundary surface are

$$V_1 = 1 \quad V_2 = 0 \quad V_3 = 0 \quad \text{at} \quad y = 0 \;\; x = 0$$

$$V_1 = 0 \quad V_2 = 1 \quad V_3 = 0 \quad \text{at} \quad y = 0 \;\; x = l_2$$

$$V_1 = 0 \quad V_2 = 0 \quad V_3 = 1 \quad \text{at} \quad y = l_1 \;\; x = l_2$$

$$V_1 = 0 \quad V_2 = 0 \quad V_3 = 1 \quad \text{at} \quad y = l_1 \;\; x = 0$$

$$V_1 = 0 \quad V_2 = 0 \quad V_3 = 1 \quad \text{at} \quad y = l_1 \;\; x = l_2/2$$

From above volume fraction distribution it is clear that the proposed volume fractions of the composition of 2D-FGM changes from 100% of V_3, material 3, on the upper surface to 100% of V_1, material 1, on the left lower corner of the lower surface and 100% of V_2, material 2, on the right corner of the lower surface. Therefore one may say that 2D-FGM is a mixture of ceramics and metals fabricated in such a way that the volume fractions of the constituents are varied continuously in a predetermined composition profile. Two different profiles for the composition of 2D-FGM are available. The first profile of 2D-FGM composition changes through the thickness from 100% ceramics at upper surface to an FGM of two different metals on the lower surface. The FGM of two different metals also changes from 100% of the first metal at left corner to 100% of the second metal at the right corner of the lower surface of the plate. The second profile of the 2D-FGM composition changes through the thickness from 100% metal at upper surface to an FGM of two different ceramic materials at the lower surface of the structure. FGM of two different ceramic materials also changes from 100% of the first ceramic material at the left corner to 100% of the second ceramic material at the right corner of the lower surface. The composition variations of the 2D-FGM through the plate has different distributions depending upon the non-homogenous parameters m_y and m_x.

4. Rules of mixture of 2D-FGM

The rules of mixture for the 2D-FGM with porosity can be obtained using the same way used to obtain the porosity of 2D-FGM with some mathematical manipulation. For any point on the 2D-FGM plate with volume fractions V_1, V_2 andV_3 as shown in Fig. 1, using Eqs. (3)–(19), the rules of mixture for the different thermal and mechanical properties may be obtained as:

For Poisson's ratio:

$$v = v_1 V_1 + v_2 V_2 + v_3 V_3 \tag{20}$$

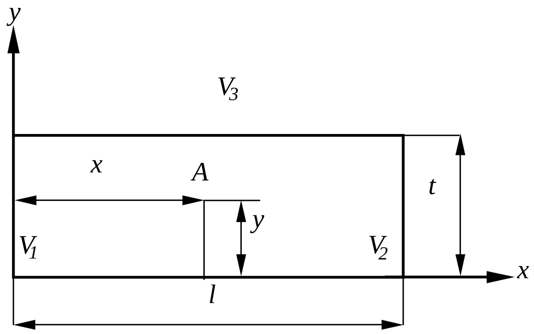

Fig. 1. Coordinate system and volume fraction distribution for 2D-FGM plate.

For modulus of elasticity

$$E = \frac{E_{0y}(1-p_y)}{1+p_y(5+8v)(37-8v)/\{8(1+v)(23+8v)\}} \tag{21}$$

where

$$E_{0y} = E_x\left[\frac{E_x+(E_3-E_x)V_3^{2/3}}{E_x+(E_3-E_x)(V_3^{2/3}-1+V_x)}\right]$$

$$E_x = \frac{E_{0x}(1-p_x)}{1+p_x(5+8v_x)(37-8v_x)/\{8(1+v_x)(23+8v_x)\}}$$

$$E_{0x} = E_1\left[\frac{E_1+(E_2-E_1)V_2^{2/3}}{E_1+(E_2-E_1)(V_2^{2/3}-V_2)}\right]$$

$$v_x = v_1V_1 + v_2V_2$$

For heat conductivity

$$\lambda = \frac{\lambda_{0y}\left\{\left(1-p_y^{2/3}\right)\lambda_y + p_y^{2/3}\lambda_a\right\}}{p_y^{1/3}\lambda_a + \left(1-p_y^{1/3}\right)\left\{\left(1-p_y^{2/3}\right)\lambda_{0y}+p_y^{2/3}\lambda_a\right\}} \tag{22}$$

where

$$\lambda_{0y} = \lambda_x\left[1+\frac{3(\lambda_3-\lambda_x)V_3}{3\lambda_xV_3+(\lambda_3+2\lambda_x)(V_2+V_1)}\right]$$

$$\lambda_x = \frac{\lambda_{0x}\left\{\left(1-p_x^{2/3}\right)\lambda_{0x}+p_x^{2/3}\lambda_a\right\}}{p_x^{1/3}\lambda_a+\left(1-p_x^{1/3}\right)\left\{\left(1-p_x^{2/3}\right)\lambda_{0x}+p_x^{2/3}\lambda_a\right\}}$$

$$\lambda_{0x} = \lambda_1\left[1+\frac{3(\lambda_2-\lambda_1)V_2}{3\lambda_1V_2+(\lambda_2+2\lambda_1)V_1}\right]$$

For coefficient of thermal expansion

$$\alpha = \alpha_x(V_1 + V_2) + \alpha_3 V_3 + \frac{V_3(V_1 + V_2)(\alpha_3 - \alpha_x)(K_3 - K_x)}{K_x(V_1 + V_2) + K_3 V_3 + (3K_3K_x/4G_x)} \tag{23}$$

where

$$\alpha_x = \alpha_1 V_1 + \alpha_2 V_2 + \frac{V_2 V_1(\alpha_2 - \alpha_1)(K_2 - K_1)}{K_1 V_1 + K_2 V_2 + (3K_2K_1/4G_1)}$$

$$K_x = K_1 V_1 + K_2 V_2$$

$$G_x = G_1 V_1 + G_2 V_2$$

For density

$$\rho = \rho_0(1 - p_t) + \rho_a p_t \tag{24}$$

where

$$\rho_0 = \rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3$$

For heat capacity

$$C = C_0(1 - p_t) + C_a p_t \tag{25}$$

where

$$C_0 = \frac{C_1 V_1 \rho_1 + C_2 V_2 \rho_2 + C_3 V_3 \rho_3}{V_1 \rho_1 + V_2 \rho_2 + V_3 \rho_3}$$

$$K_i = \frac{E_i}{3(1 - 2v_i)}$$

$$G_i = \frac{E_i}{2(1 + v_i)}$$

For yield stress

$$\sigma_Y = \sigma_{Y1} V_1 + \sigma_{Y2} V_2 + \sigma_{Y3} V_3 \tag{26}$$

The obtained rules of mixture for the 2D-FGM was adopted to obtain the variations of the thermal and mechanical properties of a 2D-FGM plate. The basic constituents materials of the 2D-FGM plate are Al1100, material 1, Ti-6Al-4V, material 2, and SiC, material 3. It was assumed that the composition of the 2D-FGM have linear variation in y-direction and non-linear variation in x-direction with non-homogenous parameter $m_x = 0.3$. The parameters n_x and z_x are considered to be equal to m_x. Also, n_y and z_y are considered to be equal to m_y. According to Bhushaan and Gupta (1997) the temperature independent thermal and mechanical material properties of the considered basic constituents are given in Table 1. The variations of the thermal and mechanical properties through the 2D-FGM plate using the obtained rules of mixture were shown in Fig. 2. It is clear that the 2D-FGM plate have non-homogeneity of the thermal and mechanical properties in x- and y-directions.

Table 1
Thermal and mechanical materials properties of FGM constituents

Material	E, GPa	ν	λ, W/m K	α, 1/K	C, J/kg K	ρ, kg/m^3	σY, MPa	Tensile strength, MPa	Compressive strength, MPa
SiC	440	0.17	100	4.3×10^{-6}	710	3210	–	175	1400
Al1100	69	0.34	220	23.6×10^{-6}	917	2715	150	–	–
Ti-6Al-4V	115	0.293	6.0	8.0×10^{-6}	610	4515	1030	–	–

5. Transient thermal loading

According to Fuchiyama et al. (1993) and Kokini et al. (1997) the crack in FGM plate may not open or initiate during the heating stage while it may initiate and propagate during the cooling stage after heating. Therefore, in the current study it will be more realistic to investigate the considered 2D-FGM under cooling thermal loading conditions. According to Steinberg (1986) and Callister (2001) it will be more realistic if the plate has temperature variations along the upper surface. Therefore, the ceramic, upper, surface of the plate ($y = t$) is considered to have initial temperature variation that shown in Fig. 3. The temperatures were changed from 1000 to 2000 K along the upper surface of the plate. The lower surface of the plate ($y = 0$) is kept at initial temperature of 300 K. Then the ceramic, upper, surface is subjected to sudden cooling to 300 K.

The initial temperature distribution over the plate was obtained by solving the considered thermal problem as steady state one. Sudden cooling of the upper surface of the plate from the initial temperature, shown in Fig. 3, to 300 K is considered using convection heat transfer on the upper surface. The transient temperature distribution for thermally shocked non-homogenous plate may be obtained using finite element method by solving the heat conduction equation for the transient state, Eq. (27).

$$\frac{\partial\lambda}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial\lambda}{\partial y}\frac{\partial T}{\partial y}+\lambda\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}\right)=\rho c\frac{\partial T}{\partial t} \tag{27}$$

The thermal boundary conditions during cooling process on the upper, ceramic, surface ($x = l$) and lower, composite metallic, surface ($x = 0$) are;

$$-\lambda\frac{\partial T}{\partial y}\bigg|_{y=t}=h(T_{y=t}-T_\infty) \tag{28}$$

$$T_{y=0}=300\ \mathrm{K} \tag{29}$$

A convection heat transfer coefficient $h = 1000$ W/m^2 K is considered due to the calculation of Choules and Kokini (1996) and Kokini et al. (1997).

6. Finite element model

A 2D-FGM plate having 15 mm thickness and 300 mm length was modeled using 2D eight-node thermal solid element. Due to the inhomogeneity of the material in x- and y-directions, every element on the finite element mesh was assigned by its own thermal and mechanical properties according to the volume fractions and the rules of mixture of 2D-FGM, Eqs. (12)–(26). The plate was initially kept at the temperature distributions generated from the following initial boundary conditions. The upper surface of the plate was initially at the temperature distribution shown in Fig. 3 while the lower surface of the plate was initially at 300 K. Then the upper surface of the plate is suddenly exposed to a cooling convection. The temperature

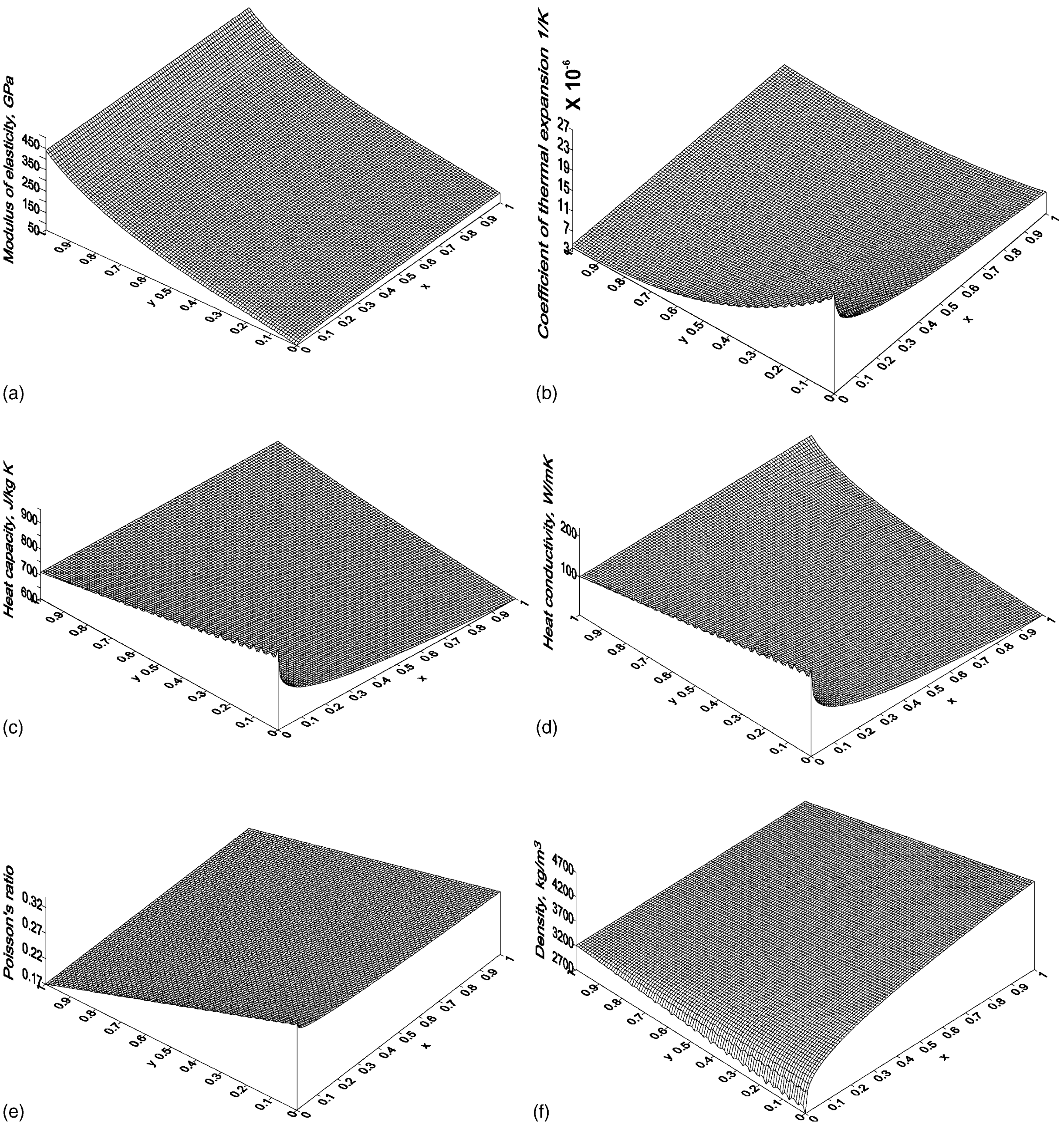

Fig. 2. Variations of the thermal and mechanical properties through the 2D-FGM plate. (a) Modulus of elasticity, (b) coefficient of thermal expansion, (c) heat capacity, (d) heat conductivity, (e) Poisson's ratio, (f) density.

solution was firstly obtained at discrete time increments by the solution of the thermal problem. The temperature distribution is then used to calculate the corresponding displacements and thermal stresses under plane strain conditions. Throughout the analysis, the node at the left lower corner of the plate, $x = 0$

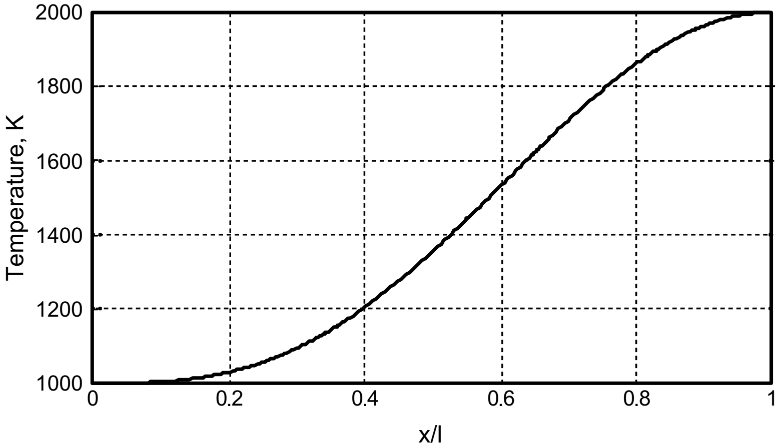

Fig. 3. Initial temperature variations through the upper surface of the 2D-FGM plate before the thermal shock.

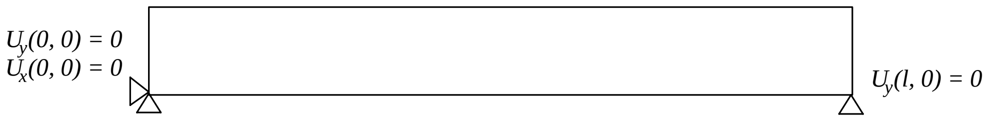

Fig. 4. Mechanical boundary conditions.

and $y = 0$, was constrained against displacement in x- and y-directions. Also, the node of the right lower corner, $x = 300$ and $y = 0$, was constrained against displacement in y-direction as shown in Fig. 4.

7. Results and discussions

Since the main advantage of the 2D-FGM over the conventional FGM is that it has variations of the compositions in two directions by adding a new material that add more strength and consequently can give more reduction to the thermal stresses or make a delay to yielding. Therefore, for proper comparison between the 2D-FGM and conventional FGM both of them are investigated under the same thermal loading. The conventional FGM constituent materials are SiC and Al1100 with linear variation in y-direction, optimum variation of the composition according to Noda (1997). The constituent materials of the 2D-FGM plate are the same as conventional FGM, SiC and Al1100, in addition to Ti-6Al-4V. The considered 2D-FGM have linear variation in y-direction, optimum variation of the composition of the conventional FGM, and non-linear variation in x-direction with non-homogenous parameter $m_x = 0.3$. The parameters n_x and z_x are considered to be equal to m_x and n_y and z_y are considered to be equal to m_y. The thermal and mechanical material properties of the constituent materials are considered to be temperature independent with the values given in Table 1.

Since the surface cracks initiation are due to the stresses parallel to surface. Therefore, the current analysis was focused on the investigation of the stresses parallel to the surface of the plate. Also, internal cracks may initiate due to the stresses parallel and normal to the surface of the plate. The transient thermal stresses problem was solved for both of the 2D-FGM and the conventional FGM plates. Then for proper comparison the values of the thermal stresses was normalized by the values of the yield stresses. Where the yield stress σ_Y is calculated from Eqs. (26) and (12) for 2D-FGM and the conventional FGM respectively. It is noteworthy that for SiC the tensile and compressive strength are used instead of the yield stress according to the type of the stresses, compressive or tensile. Also, the normalized compressive stresses are represented by negative values.

From the current investigations it was found that the maximum values of the thermal stresses in y-direction, σ_y, were of negligible values along the plate except for the right and lift vertical surfaces of the plate. The normalized values of the maximum stresses in y-direction, σ_y/σ_Y, are 0.216 for 2D-FGM and 0.54 for conventional FGM, respectively. This means that yielding will not occurs due to thermal stresses in y-direction. Also, it is found that the severe values of the thermal stresses were in x-direction, σ_x. Therefore the thermal stresses in x-direction are of prime importance.

Fig. 5 shows the variations of the maximum and minimum normalized stresses in x-direction, σ_x, $(\sigma_x)_{\max}/\sigma_Y$ and $(\sigma_x)_{\min}/\sigma_Y$ versus the cooling time for both of the 2D-FGM and the conventional FGM plates. From Fig. 5 it is clear that, for the conventional FGM, most values of the maximum normalized thermal stresses in x-direction are greater than unity with maximum value of 1.6, which occurs just at the beginning of applying sudden cooling. This means that the cracks initiate just at the time of applying the sudden cooling or yielding occurred in the conventional FGM under the considered thermal loading. For the 2D-FGM all values of the maximum normalized thermal stresses in x-direction are less than unity with maximum value of 0.516. Which means that yielding or crack initiation of the 2D-FGM plate does not occur under the considered thermal loading. Also, one can notice that all the values of the minimum, maximum compressive, normalized thermal stresses for both of the 2D-FGM and conventional FGM plates are less than unity.

Figs. 6 and 7 show the variations of the normalized thermal stresses in x-direction for the 2D-FGM and conventional FGM plates at the time during which maximum values of the normalized thermal stresses occurred, 2.001 and 0.001 s for 2D-FGM and conventional FGM respectively. From Fig. 6 it is clear that all the values of the normalized thermal stresses, tension and compression, through the 2D-FGM plate are less than unity, the yield value. The maximum value of the normalized thermal stresses is 0.516 that occurs at $x/l = 0.96$ and $y/t = 0$, close to the right side of the lower surface of the plate. The upper surface of the

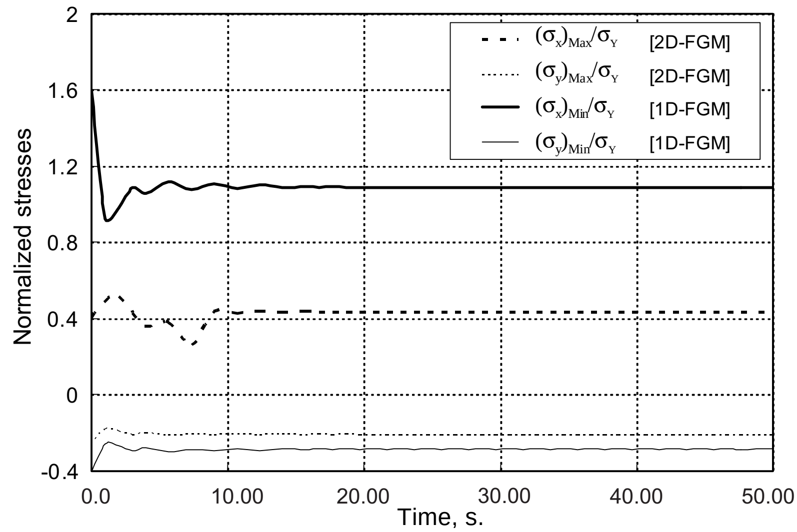

Fig. 5. Variations of the normalized maximum and minimum thermal stresses in x-direction versus the cooling time for the 2D-FGM and conventional FGM plates.

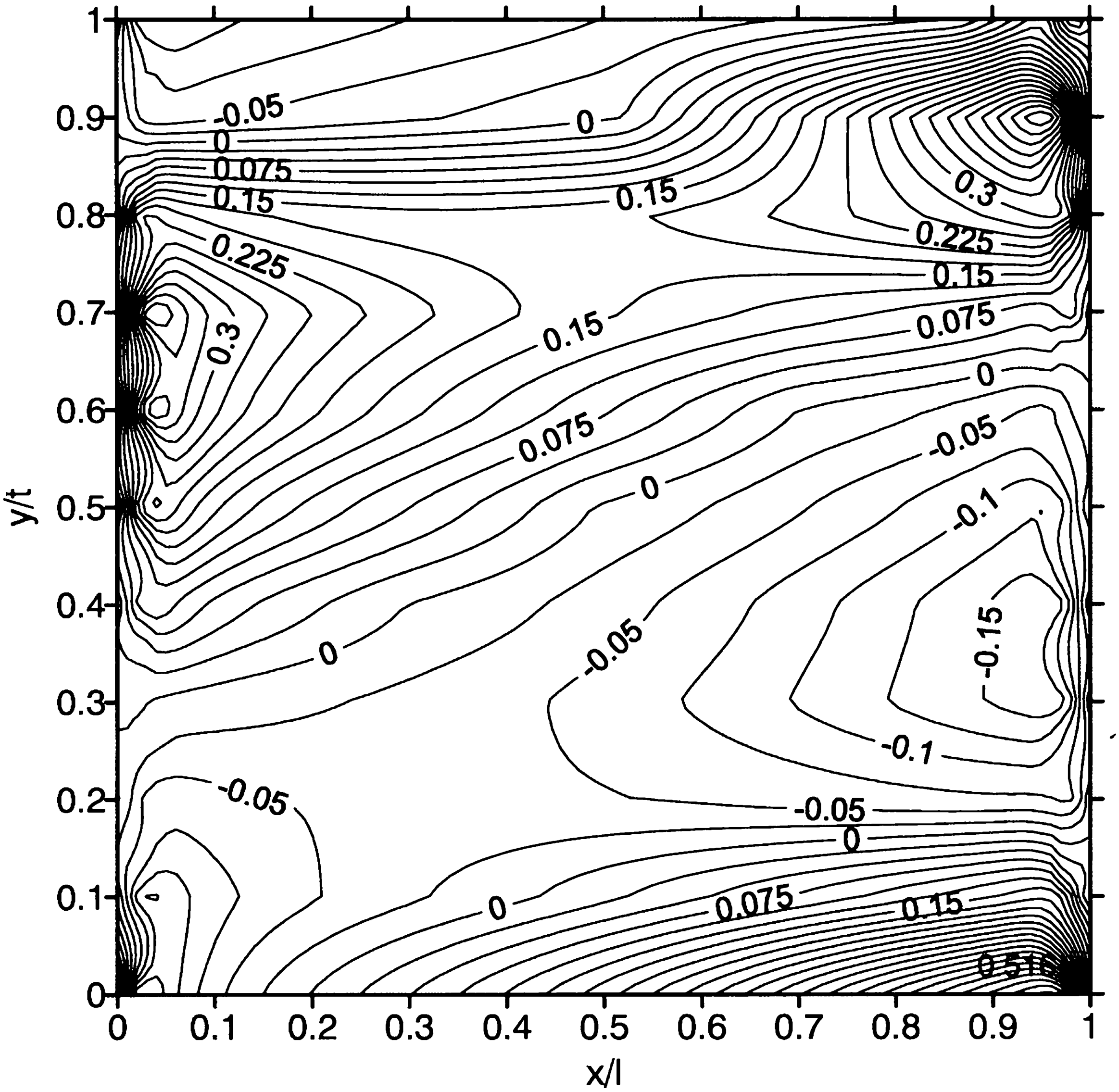

Fig. 6. Variations of the normalized thermal stresses in x-direction for the 2D-FGM plate at the time during which maximum values of the normalized thermal stresses were occurred, 2.001 s.

plate is subjected to a compressive thermal stresses except the right corner which is subjected to tensile thermal stresses.

From Fig. 7 it is clear that for conventional FGM yielding occurred in the region $0.62 > y/t > 0.835$ in addition to the right corner of the lower surface of the plate, which means that cracks initiate in these regions. The maximum value of the normalized thermal stresses is 1.6 that occurs at $x/l = 0.94$ and $y/t = 0.767$. The upper surface of the plate is subjected to a compressive thermal stresses.

From Figs. 6 and 7 it is clear that the 2D-FGM is capable of bearing the thermal stresses without yielding than the conventional FGM under the same thermal loading conditions. Also, the position of the

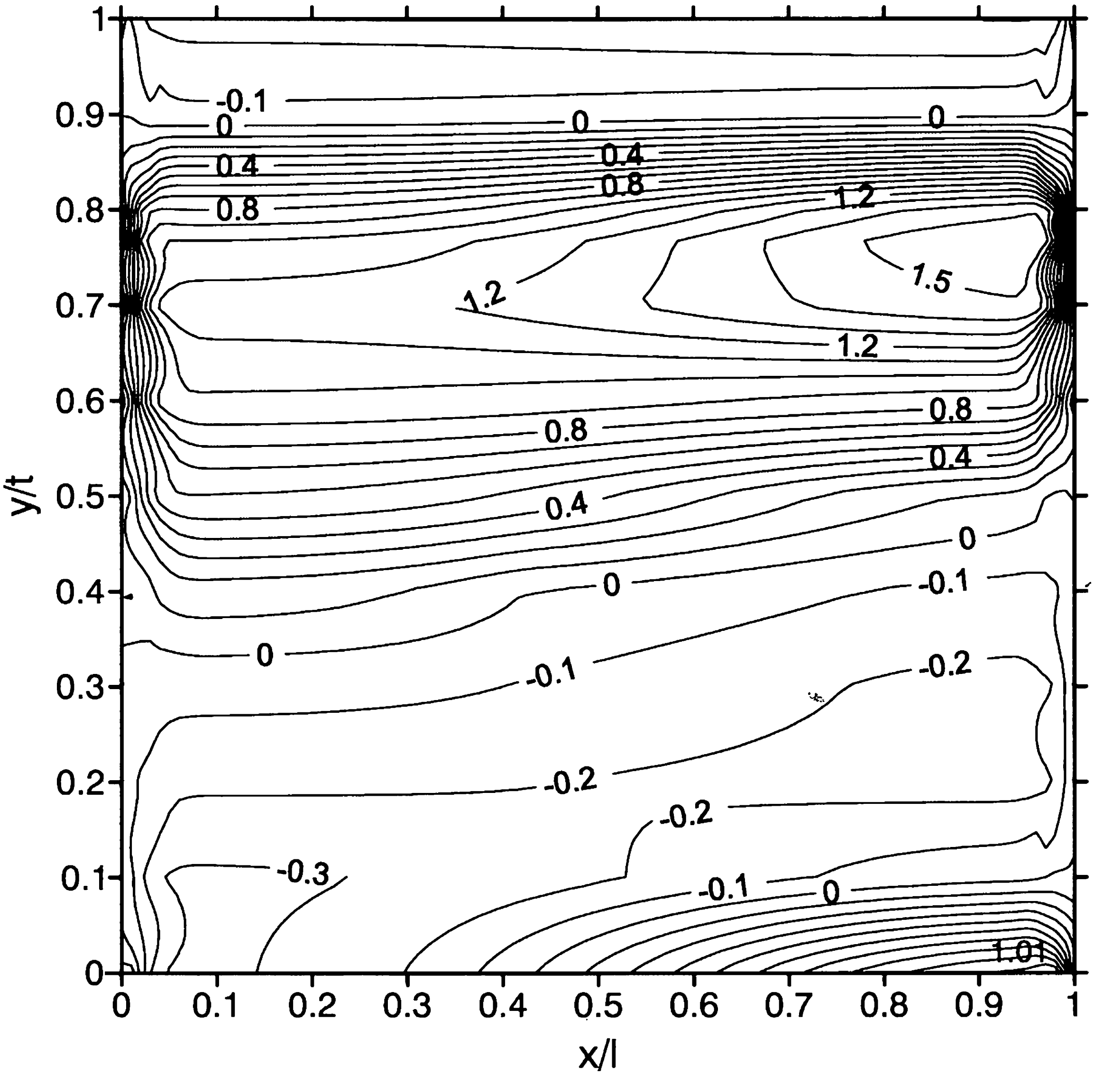

Fig. 7. Variations of the normalized thermal stresses in x-direction for the conventional FGM plate at the time during which maximum values of the normalized thermal stresses occurred, 0.001 s.

maximum thermal stresses was shifted in the 2D-FGM to the right corner of metallic side, lower surface that have high strength. While in the conventional FGM it was near to the ceramic side, upper surface, direction of decreasing mechanical strength. This is also an advantage for the 2D-FGM over the conventional FGM where the metallic side always has high mechanical strength. It is also clear that the zone of maximum normalized thermal stresses in 2D-FGM is smaller than that of conventional FGM.

Figs. 8 and 9 show the variation of the residual stresses, in x-direction, in both of the 2D-FGM and conventional FGM plates respectively. Residual stresses are the thermal stresses remaining in the plate after

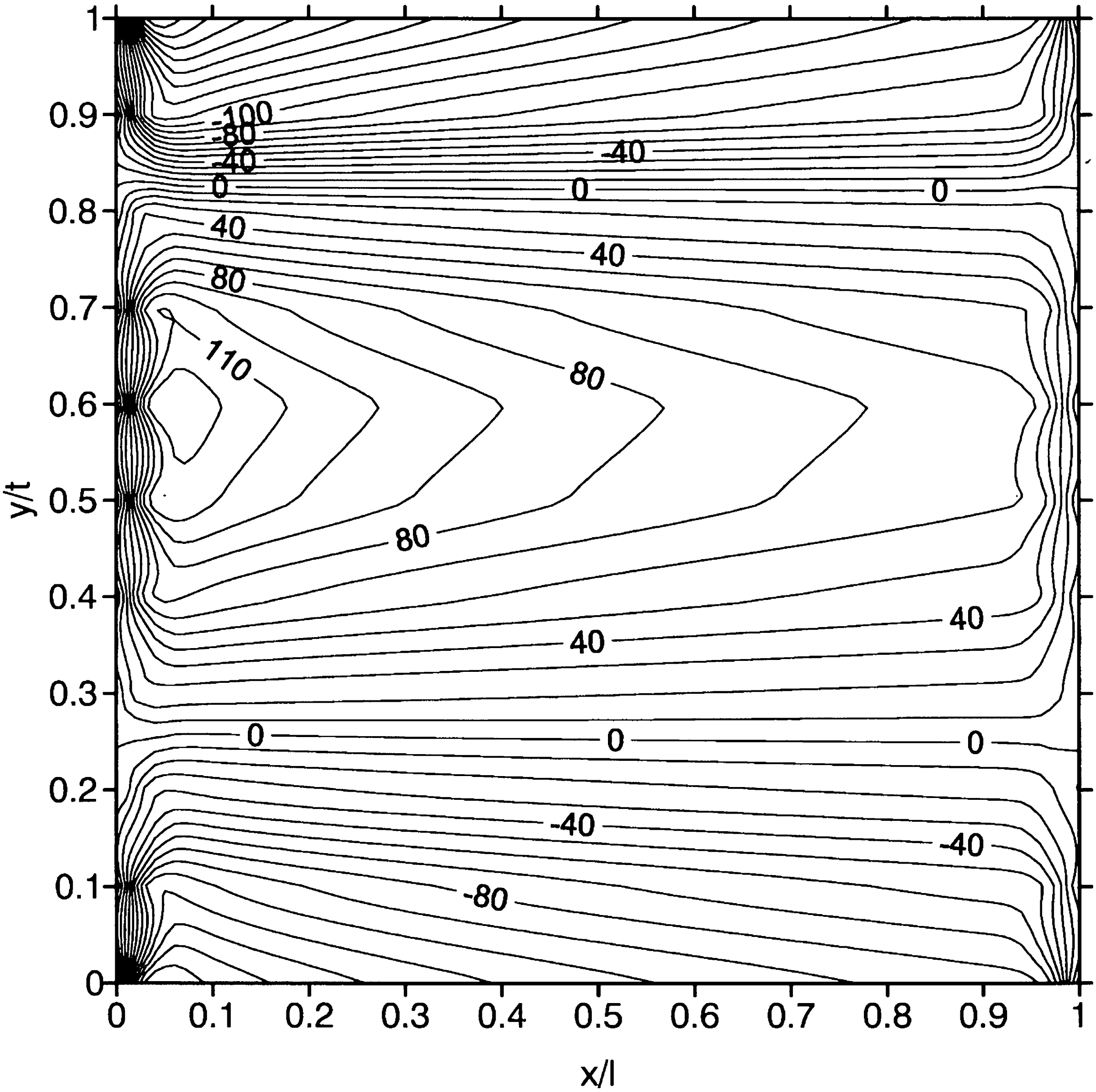

Fig. 8. Variations of the residual stresses in x-direction for the 2D-FGM plate, MPa, after 100 s.

ending of the cooling processes, 100 s. It is clear that the upper and lower surface of the plate in both cases has compression residual stresses. Values of compressive residual stresses in upper and lower surfaces of the plate are higher for the conventional FGM than the 2D-FGM. The distribution of the residual stresses in the conventional FGM is uniform, in x-direction, than the 2D-FGM. This is an expected result due to the non-homogeneity in x- and y-direction of the 2D-FGM. The position of the maximum residual stresses is the same in the two cases, $y/t = 0.6$, but the 2D-FGM has lower value, 125 MPa, than the conventional FGM, 169.7 MPa. Zone of maximum residual stresses is smaller for 2D-FGM than the conventional FGM.

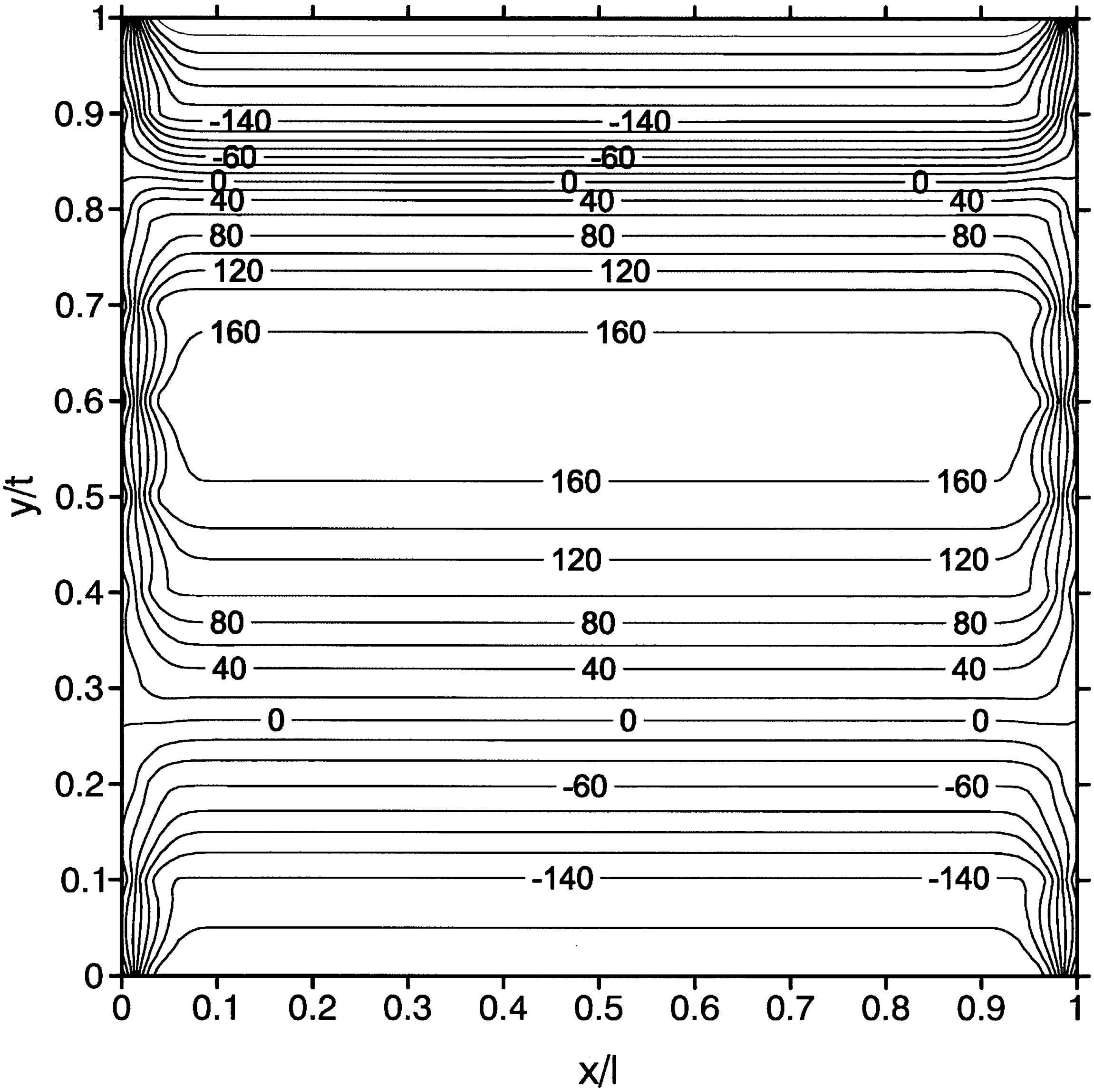

Fig. 9. Variations of the residual stresses in x-direction for the conventional FGM plate, MPa, after 100 s.

8. Conclusions

Through the current investigation for the introduced 2D-FGM the volume fractions are proposed and rules of mixture are obtained. The volume fractions and rules of mixture for the introduced 2D-FGM were used to calculate the thermal stresses in SiC/Al1100/Ti-6Al-4V 2D-FGM plate, with temperature independent material properties. The SiC/Al1100/Ti-6Al-4V 2D-FGM plate was subjected to steady thermal load that has temperature variations along the ceramic surface, SiC. Then the 2D-FGM plate was subjected to transient cooling condition. Comparison between SiC/Al1100/Ti-6Al-4V 2D-FGM, with linear composition vitiation in y-direction, optimum composition of the conventional FGM, and non-linear variation in x-direction with non-homogenous parameter $m_x = 0.3$, and conventional SiC/Al1100 FGM, with linear composition variation in y-direction, showed that 2D-FGM has high capabilities to reduce thermal and residual stresses than conventional FGM. Under the same thermal loading conditions, yielding occurred for conventional SiC/Al1100 FGM while SiC/Al1100/Ti-6Al-4V 2D-FGM did not. Also, zone of maximum thermal stresses is smaller for 2D-FGM than conventional FGM.

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